HOMEWORK 3

Due November 7, 2022

1. Find the transfer function of a Chebyshev lowpass filter with the following specifications:

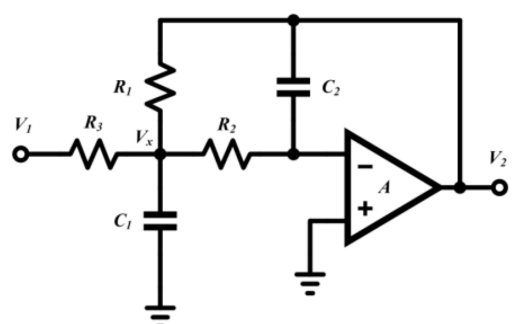
Passband 0-20 kHz, ripple 0.3 dB

Stopband over 50 kHz, gain below -22 dB

Suggest a circuit realizing this filter. It may contain passive as well as active stages.

2. Find the transfer function of the Rauch filter. What is the dc gain? What are the zeros and poles? What is the pole Q?

   Start of solution: use KCL to find the transfer function:



Transfer function:

$$H(s) = \frac{G_2 G_3 /(C_1 C_2)}{s^2 + s(G_1 + G_2 + G_3)/C_1 + G_1 G_2 /(C_1 C_2) + \varepsilon}$$

1.

Given: $f_P = 20kHz$, $\alpha_P = 0.3dB$;

$f_S = 50kHz$, $\alpha_S = 22dB$;

Selective parameter: $k = \frac{f_P}{f_S} = \frac{20kHz}{50kHz} = 0.4$

Discrimination parameter: $k_1 = \sqrt{\frac{10^{\frac{\alpha_P}{10}}-1}{10^{\frac{\alpha_S}{10}}-1}} = \sqrt{\frac{10^{0.03}-1}{10^{2.2}-1}} = 0.3293$

For Chebyshev filter: $n \geq \frac{\cosh^{-1}\frac{1}{k_1}}{\cosh^{-1}\frac{1}{k}} = \frac{4.5416}{1.5668} \approx 2.899$

Hence, order n is chosen as 3. Then

$$\cos 3u = 4\cos^3 u - 3\cos u = 4\Omega^3 - 3\Omega$$

$$|K|^2 = k_P^2(4\Omega^3 - 3\Omega)^2$$

where $k_P = \sqrt{10^{\frac{\alpha_P}{10}} - 1} \approx \sqrt{10^{0.03} - 1} \approx 0.2674$

$$K(S) = \pm(4S^3 - 3S) \approx \pm(1.0697S^3 - 0.8023S)$$

Natural modes: $v_k = \pm\frac{\left(k-\frac{1}{2}\right)\pi}{3}$ where k=1,2,3 and $w_k = \pm\frac{1}{3}\sinh^{-1}\frac{1}{k_P} \approx 0.3485$

Therefore, the 3 natural modes are

$\Sigma_1 = -\sin\frac{\pi}{6}\sinh w_1 \approx -0.3647$, $\Omega_1 = \cos\frac{\pi}{6}\cosh w_1 \approx 1.0719$

$\Sigma_2 = -\sin\frac{\pi}{2}\sinh w_2 \approx -0.7293$, $\Omega_1 = \cos\frac{\pi}{2}\cosh w_2 = 0$

$\Sigma_3 = -\sin\frac{5\pi}{6}\sinh w_3 \approx -0.3647$ $\Omega_1 = \cos\frac{5\pi}{6}\cosh w_3 \approx -1.0719$

Zeros: 3 at infinity; poles: $-0.7293, -0.3647 \pm j1.0719$

$$H(S) = \pm 2^{3-1} k_P (S^2 - 2\Sigma_1 S + \Sigma_1^2 + \Omega_1)(S - \Sigma_2)$$
$$= \pm C(S^3 + a_2 S^2 + a_1 S + a_0)$$

$$2^{3-1} * 0.2674(S^2 + 2 * 0.3647S + 0.3647^{\ 2} + 1.0719^2)(S + 0.7293)$$
$$= 1.0697(S^3 + 1.4587S^2 + 1.8139S + 0.9349)$$

$$A(S) = \frac{1}{H(S)} = \frac{1}{1.0697S^3 + 1.5604S^2 + 1.9403S + 1}$$

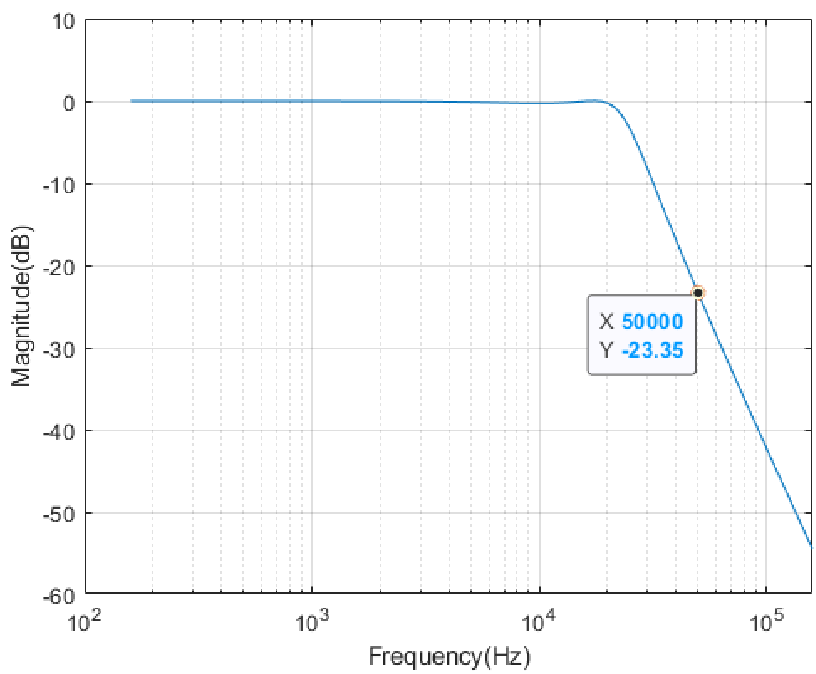
Since $\omega_0 = 2\pi f_P = 2\pi * 20 * 10^3 = 4\pi * 10^4$

$$A'(S) = A\left(\frac{S}{\omega_0}\right) = \frac{\omega_0^3}{1.0697S^3 + 1.5604\omega_0 S^2 + 1.9403\omega_0^2 S + \omega_0^3}$$
$$= \frac{C_0}{C_3 S^3 + C_2 S^2 + C_1 S + C_0}$$

$C_3 = 1.0697$, $C_2 = 1.9609 * 10^5$, $C_0 = 3.0640 * 10^{10}$, $C_0 = 1.9844 * 10^{15}$

$$A'(S) = \frac{1.9844 * 10^{15}}{1.0697S^3 + 1.9609 * 10^5 S^2 + 3.0640 * 10^{10} S + 1.9844 * 10^{15}}$$
$$= \frac{1}{S^3 + 1.8331 * 10^5 S^2 + 2.8644 * 10^{10} S + 1.8551 * 10^{15}}$$

Magnitude response for this transfer function:



For $f = 50kHz$, $S = \frac{j\omega}{\omega_0} = j2.5$

$$A(j2.5) \approx \frac{1}{1.0697S(j2.5)^3 + 1.5604(j2.5)^2 + 1.9403(j2.5) + 1} \approx \frac{1}{-8.7525 - j11.8633}$$

$$|A(j2.5)| \approx \frac{1}{14.74} \approx -23.4dB$$

This can be implemented as a Rauch Biquad filter cascaded with a passive lowpass RC filter.

$$A'(S) = \frac{\omega_0^3/1.0697}{(s-p_1)(s-p_2)(s-p_3)} = \frac{p_1 * p_2}{(s-p_1)(s-p_2)} \frac{-p_3}{(s-p_3)}$$

Biquad Poles: $p_{1,2} = (-0.3647 \ \pm j1.0719)\omega_0$; Poles: $-0.7293\omega_0$

(1) Biquad stage: $H_1(s) = \frac{p_1 p_2}{(s-p_1)(s-p_2)}$

-$(p_1 + p_2) = 2 * 0.3647\omega_0 = \frac{G_1+G_2+G_3}{C_1}$, $p_1 * p_2 = (0.3647^2 + 1.0719^2)\omega_0^2 = \frac{G_1 G_2}{C_1 C_2}$

(2) RC stage: $H_2(s) = \frac{-p_3}{s-p_3}$, $\frac{1}{s+0.7293\omega_0} = \frac{1}{sR'C'+1}$

$$\frac{1}{R'C'} = 0.7293\omega_0$$

2.

The dc gain is G_3/G_1. There are two zeros at infinite frequencies. The poles are the zeroes s_p and s_p* of the denominator. To find the relation between the coefficients a_i, and the pole frequency and pole Q, rewrite the denominator:

$s^2 + a_1 s + a_0 = (s - s_p)(s - s_p^*) = s^2 + (|s_p|/Q)s + |s_p|^2$

Hence, $|s_p| = [G_1G_2/C_1C_2]^{1/2}$, and $Q = |s_p|/a_1$.